Week 9 - Friday

COMP 2230

Last time

- Finished pigeonhole principle
- Combinations
- Combinations with replacement
- Started binomial theorem

Questions?

Assignment 4

Logical warmup

- Draw a (non-self-intersecting) hexagon that can be cut into four congruent triangles by a single line
- **Hint:** Try to make it out of right triangles

Binomial Matters

Pascal's Triangle

- The beginning of Pascal's Triangle is:

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

- If we number rows and columns starting at 0, note that the value of row n, column r is exactly $\binom{n}{r}$

Binomial theorem

- $a + b$ is called a **binomial**
- Using combinations (or Pascal's Triangle) gives an easy way to compute $(a + b)^n$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

- We could prove this by induction, but you probably don't care

Binomial example

- Compute $(1 - x)^6$ using the binomial theorem
- Now try $(6a + 2b)^7$

Probability Axioms and Expected Value

Probability axioms

- Let A and B be events in the sample space S
  - $0 \leq P(A) \leq 1$
  - $P(\emptyset) = 0$ and $P(S) = 1$
  - If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$
  - It is clear then that $P(A^c) = 1 - P(A)$
  - More generally, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- All these axioms can be derived from set theory and the definition of probability

Union probability example

- What is the probability that a card drawn randomly from an Anglo-American 52 card deck is a face card (jack, queen, or king) or is red (hearts or diamonds)?
- Hint:
  - Compute the probability that it is a face card
  - Compute the probability that it is red
  - Compute the probability that it is both

Expected value

- **Expected value** is one of the most important concepts in probability, especially if you want to gamble
- The expected value is simply the sum of all events, weighted by their probabilities
- If you have n outcomes with real number values $a_1, a_2, a_3, \dots\ a_n$, each of which has probability $p_1, p_2, p_3, \dots\ p_n$, then the expected value is:

$$\sum_{k=1}^{n} a_k p_k$$

Expected value: Roulette

- A normal American roulette wheel has 38 numbers: 1 through 36, 0, and 00
- 18 numbers are red, 18 numbers are **black**, and 0 and 00 are green
- The best strategy you can have is always betting on **black** (or red)
- If you bet $1 on **black** and win, you get $1, but you lose your dollar if it lands red
- What is the expected value of a bet?

Conditional Probability

Three-Sentence Summary

Conditional Probability

Conditional probability

- Given that some event A has happened, the probability that some event B will happen is called conditional probability
- This probability is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Conditional probability example

- Given two, fair, 6-sided dice, what is the probability that the sum of the numbers they show when rolled is 8, given that both of the numbers are even?

Bayes' Theorem

- Let sample space S be a union of mutually disjoint events $B_1, B_2, B_3, \dots\ B_n$
- Let A be an event in S
- Let A and B_1 through B_n have non-zero probabilities
- For B_k where $1 \leq k \leq n$
- $P(B_k|A) =$

$$\frac{P(A|B_k) \cdot P(B_k)}{P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2) + \dots + P(A|B_n) \cdot P(B_n)}$$

Applying Bayes' theorem

- Bayes' theorem is often used to evaluate tests that can have false positives and false negatives
- Consider a test for a disease that 1 in 5,000 people have
  - The false positive rate is 3%
  - The false negative rate is 1%
- What's the probability that a person who tests positive for the disease has the disease?
- Let A be the event that the person tests positively for the disease
- Let B_1 be the event that the person actually has the disease
- Let B_2 be the event that the person does not have the disease
- Apply Bayes' theorem

Independent events

- If events A and B are events in a sample space S, then these events are independent if and only if
$$P(A \cap B) = P(A) \cdot P(B)$$
- This should be clear from conditional probability
- If A and B are independent, then $P(B|A) = P(B)$

$$P(B|A) = P(B) = \frac{P(A \cap B)}{P(A)}$$

$$P(A) \cdot P(B) = P(A \cap B)$$

Upcoming

Next time…

- Graphs: trails, paths, and circuits
- **No class next week!**

Reminders

- **Finish Assignment 4**
  - **Due tonight by midnight!**
- Read 10.1
  - Prepare a three-sentence summary
  - Extra credit if you get called on